Seat No. ________________

H-003-2016001

B. Sc. (Sem. VI) (CBCS) (W.E.F. 2019) Examination

April - 2023

Math - Paper - 08 (A)

(Graph Theory & Complex Analysis - II)

Faculty Code : 003
Subject Code : 2016001

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions : Figures to the right indicates full marks of the questions.

1 (a) Answer the following questions : **4**

(1) Define: Degree of a vertex in a graph.

(2) The number of edges in K_9 is ________.

(3) An undirected graph G has 8 edges. Find the number of vertices, if the degree of each vertex in G is 2.

(4) Define : Binary tree.

(b) Attempt any **one** : **2**

(1) Prove that if a graph has exactly two vertices of odd degree, then there must be a path joining these two vertices.

(2) Prove that number of vertices in a binary tree is always odd.

(c) Attempt any **one** : **3**

(1) Prove that if n is odd, then the number of edge-disjoint Hamiltonian cicuits in K_n is $\frac{n-1}{2}$.

(2) Prove that a graph is a tree if and only if it is minimally connected.

(d) Attempt any **one** : **3**

(1) Prove that the number of edges in a tree with n vertices is $n - 1$.

(2) Prove that a graph G is Euler graph if and only if every vertex of G is of even degree.

2 (a) Answer the following questions : **4**

(1) If G is a planar graph and G^* is dual of G, then rank of G = _______ of G^*.

(2) Define: Vertex connectivity of a graph.

(3) For a connected graph G with 5 vertices and 8 edges, what is the dimension of circuit subspace of a vector space associated with G?

(4) What is the number of faces in a connected planar graph with 5 vertices and 7 edges?

(b) Attempt any **one** : **2**

(1) Prove that chromatic number of any tree is 2.

(2) Define: Incidence matrix. Find incidence matrix of K_4.

(c) Attempt any **one** : **3**

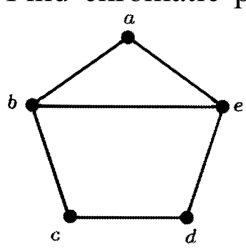
(1) Prove that rank of incidence matrix of a graph with n vertices is $n - 1$.

(2) Prove that every circuit in a graph G has an even number of edges in common with any cut-set of G.

(d) Attempt any **one** : **5**

(1) In usual notations, prove that $\left(W_G, +_2\right)$ is an abelian group.

(2) Find chromatic partitioning of the following graph.



3 (a) Answer the following questions : **4**

(1) Prove that $f(z) = e^z$ is conformal map at each point of Z plane.

(2) Write a linear transformation that reflect expansion or contraction.

(3) Define: Critical point.

(4) What is general form of a bilinear transformation?

(b) Attempt any **one** : **2**

(1) Describe the transformation $w = z + c$, where c is any complex number.

(2) Determine fixed points of the bilinear transformation $w = \dfrac{z-1}{z+1}$.

(c) Attempt any **one** : **3**

(1) Show that the linear transformation $w = \frac{1}{z}$ maps a horizontal line $y = c_2 \left(c_2 \neq 0\right)$ onto the circle $u^2 + \left(v + \frac{1}{2c_2}\right)^2 = \left(\frac{1}{2c_2}\right)^2$.

(2) Under the transformation $w = z^2$, find the image of infinite strip $1 \leq \operatorname{Re}(z) \leq 2$.

(d) Attempt any **one** : **5**

(1) Show that the mapping $(w+1)^2 = \frac{4}{z}$ maps a unit circle in w-plane into a parabola in z-plane.

(2) Show that the linear transformation $z = \frac{i-w}{i+w}$ transforms upper half plane $v \geq 0$ in w-plane onto all points within and on unit circle in z-plane.

4 (a) Answer the following questions : **4**

(1) Every convergent series is absolutely convergent. (True/False)

(2) A series $\sum_{n=0}^{\infty} \frac{1}{1-z}$ is convergent for ________.

(3) Write Maclaurin's series of sinh z.

(4) Define: Radius of convergence.

(b) Attempt any **one** : **2**

(1) $\sum z_n$ is convergent then prove that $\lim_{n \to \infty} z_n = 0$.

(2) Find the Maclaurin's series of $z \cos(z)$.

(c) Attempt any **one** : **3**

(1) Find Laurent's series expansion for the function $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $1 < |z| < 2$.

(2) Expand $\cos z$ into a Taylor's series about the point $z_0 = \frac{\pi}{2}$.

(d) Attempt any **one** : **5**

(1) What is the largest circle within which the Maclaurian series for the function tanh z converges to tanh z? Write the first two nonzero terms of that series.

(2) Derive Laurent's series of $f(z)$ about point z_0.

5 (a) Answer the following questions : **4**

(1) The value of $\int_C e^{\frac{1}{z}} dz$ is _______.

(2) What is the residue of $\frac{\sin z}{z}$ at $z_0 = 0$?

(3) Let $f(z) = \frac{1}{(z-2)^4 (z+3)^6}$. Then $z = -3$ is the pole of order _________.

(4) Define : Isolated singular point.

(b) Attempt any **one** : **2**

(1) Prove that $\int_C e^{\frac{1}{z^2}} dz = 0, C : |z| = 1$.

(2) Find residue of $\cot z$ at $z = \pi$.

(c) Attempt any **one** : **3**

(1) Discuss the types of isolated singularities with suitable examples.

(2) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is the circle $|z| = \frac{3}{2}$.

(d) Attempt any **one** : **5**

(1) Evaluate $\int_C \frac{2z+3}{z(z-1)} dz; C : |z| = 2$.

(2) Using Cauchy's residue theorem, show that

$$\int_0^{2\pi} \frac{1}{\cos\theta + 2} d\theta = \frac{2\pi}{\sqrt{3}}.$$